

III. RESULTS FOR 7 MM OPEN STANDARD

The above method was used on an open calibration standard that accompanies HP8510 network analyzers. The 7 mm open, part number MMC2616D1, was evaluated from 0.1 to 20 GHz. A piece of 10 cm air line, HP11566A with support beads, was used to offset the calibration standards.

Measurements were first made on the three standards at a reference plane. A fixed load was used between 0.1 and 2 GHz, with a sliding load used at the higher frequencies. A machine averaging factor of 1024 was used to improve measurement accuracy. These measurements were repeated a second time, in which the three standards were offset by the piece of air line.

The measured phase shift of the imperfect open is shown in Fig. 3. The raw data are presented along with a least square fit of the data. Increased averages and repeated acquisitions can improve the raw data. The true phase shift is a smooth function. The least square fit of the data yields the exact correction factor needed for the open. A detailed printout of the phase shift of the 7 mm open is available on request.

IV. APPLICATION OF METHOD

This approach was found most useful in a measurement study of coaxial discontinuities. An investigation was done to determine the axial separation distance of the inner and outer step discontinuities which produced minimum reflections. To facilitate fabrication of test pieces, a 7 mm to 14 mm type transition was studied. This required a calibration procedure using a 14 mm prototype open.

Measurements of the open's phase shift for the 14 mm open are presented in Fig. 4. Again the least square fit of the raw data was used to determine the phase shift factor. The phase shift factor was used to determine the error terms, Er and Es , needed in the measurement. A detailed printout of the phase shift for the 14 mm open is available on request.

V. CONCLUSIONS

This empirical method can be used to accurately determine the correction factor needed for imperfect opens due to fringing capacitance. It offers the advantage of determining the unknown phase shift for an open by using the same unknown open, an air line with unknown properties, and a network analyzer which is not calibrated. No other calibration kits or modeling coefficients are necessary. This method can be used with opens of any type if an accompanying short, match, and air line exist.

REFERENCES

- [1] R. Hackborn, "An automatic network analyzer system," *Microwave J.*, vol. 11, pp. 45-52, May 1968.
- [2] B. Hand, "Developing accuracy specifications for automatic network analyzer systems," *Hewlett-Packard J.*, vol. 21, pp. 16-19, Feb. 1970.
- [3] W. Kruppa, "An explicit solution for the scattering parameters of a linear two-port measured with an imperfect test set," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-19, pp. 122-123, Jan. 1979.
- [4] S. Rehnmark, "On the calibration process of automatic network analyzer systems," *IEEE Trans. Microwave Theory Tech.*, pp. 457-458, Apr. 1974.
- [5] E. DaSilva and M. McPhun, "Calibration of an automatic network analyzer using transmission lines of unknown characteristic impedance, loss and dispersion," *Radio Electron. Eng.*, vol. 48, no. 5, pp. 227-234, May 1978.
- [6] J. Fitzpatrick, "Error models for systems measurement," *Microwave J.*, pp. 63-66, May 1978.
- [7] D. Woods, "Shielded-open-circuit discontinuity capacitances of a coaxial line," *Proc. Inst. Elec. Eng.*, vol. 119-12, pp. 1691-1692, Dec. 1972.
- [8] B. Bianco, "Open circuited coaxial lines as standards for microwave measurements," *Electron. Lett.*, pp. 373-374, May 8, 1980.

An Analytical Approach to the Analysis of Dispersion Characteristics of Microstrip Lines

Dorel Homentcovschi

Abstract—A new analytical method for determining the dispersion characteristics of microstrip lines is given. The method uses dual integral equations, and the dispersion relation is obtained in terms of a double infinite system of linear equations with good convergence properties.

I. INTRODUCTION

Microstrip is one of the most important elements in microwave integrated circuits and microwave networks. In the early stage of microstrip-line analysis, much of the work was based on the quasi-TEM approximation [1]-[4]. This approximation is valid only for low frequencies, and the resulting parameters, such as characteristic impedance and the propagation wavenumber, are independent of frequency. However, this approximate model is inadequate for estimating the dispersion properties of the microstrip line at higher frequencies; consequently, a more rigorous full-wave analysis is required [5]. Various methods have been employed to calculate the dispersion characteristics of the stripline. Thus Hornsby and Gopinath [6] applied the finite difference method and a minimization technique. Daily [7] applied the finite element method; Zysman and Varon [8] formulated the integral equations of the problem; and Yamashita and Atuski [9] solved these integral equations numerically by nonuniform discretization of the integral domains. For shielded microstrip lines Mittra and Itoh [10] used the singular integral equation approach for deriving a new form of the dispersion equation with superior convergence properties. The spectral-domain approach has also often been applied to the full-wave analysis of the microstrip lines [11]-[14]. We also mention application to the microstrip problem of the variational conformal mapping technique [15].

Some of the developed methods are based on the assumption of certain "closed form" expressions for the longitudinal and transverse current distributions on the strip. As the proposed forms do not reveal the frequency and dielectric constant dependence of the current distributions with good accuracy, the results obtained with various methods have sometimes been quite different [16].

In this paper we developed a method to analyze the problem of microstrip shielded by two parallel planes similar to the method given by Mittra and Itoh [19] for the case of the completely shielded microstrip. Since in our case the dielectric domain is infinite, there follows a system of two integral equations instead of series equations corresponding to the bounded dielectric domain considered in [10]. We have succeeded in transforming the system of integral equations into an infinite system of linear equations. As a by-product, there follow two compatibility conditions which yield the dispersion equation of the problem.

II. FORMULATION OF THE PROBLEM

In Fig. 1 the cross section of the microstrip line to be analyzed is shown. The geometry contains a conducting strip placed on a dielectric substrate and two perfectly conducting planes. The

Manuscript received January 25, 1990; revised October 3, 1990.

The author is with the Faculty of Automatics, Polytechnic Institute of Bucharest, 313 Splaiul Independentei, Bucharest, Romania.

IEEE Log Number 9042493.

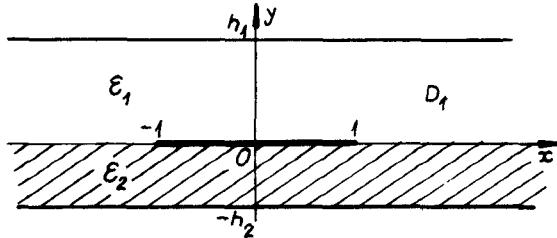


Fig. 1. Geometry of the problem.

strip conductor is assumed to be negligibly thin and lossless. We consider all the length referred to the strip length and the axis is chosen so that the strip lies on the segment $[-1, 1]$ of the x axis. Let the space over the strip be occupied by a homogeneous dielectric of relative permittivity ϵ_1 and height h_1 . The lower domain, D_2 , of height h_2 is occupied by a dielectric of relative permittivity ϵ_2 .

A hybrid-mode analysis is necessary in this inhomogeneous structure. We denote by $\psi^{(e)}$ and $\psi^{(h)}$ the scalar potentials for TM waves and TE waves, respectively. The electromagnetic (EM) fields of hybrid modes are given by the relations which can be found in [9], [10], and [17]. The two scalar potentials satisfy two-dimensional Helmholtz equations in domains D_1 and D_2 as well as the condition that the tangential components of the electric field vanish on the strip and on the two shielding planes. In view of these conditions we write

$$\psi_j^{(e)} = \int_0^\infty \frac{A(k)}{\epsilon_j - \bar{\beta}^2} \frac{\sinh \alpha_j (h_j + (-1)^j y)}{\sinh \alpha_j h_j} \cos kx dk \quad (1)$$

$$\begin{aligned} \psi_j^{(h)} = & \frac{(-1)^j \beta}{\omega \mu} \int_0^\infty \frac{k}{\alpha_j} \left(k^2 B(k) + \frac{A(k)}{\epsilon_j - \bar{\beta}^2} \right) \\ & \cdot \frac{\cosh \alpha_j (k_j + (-1)^j y)}{\sinh \alpha_j h_j} \sin kx dk. \end{aligned} \quad (2)$$

Here $\alpha_j = \sqrt{k^2 + \beta^2 - \epsilon_j k_0^2}$, $\bar{\beta} = \beta/k_0$ is the normalized propagation constant, and $A(k)$ and $B(k)$ are functions to be determined.

The EM field derived from potentials $\psi_j^{(e)}$ and $\psi_j^{(h)}$ satisfies the interface conditions at $y = 0$:

$$E_{x1} = E_{x2} \quad E_{z1} = E_{z2}, \quad x \in (-\infty, \infty). \quad (3)$$

Besides these conditions, the EM field must satisfy at the circuit plane the following relations:

$$\begin{aligned} E_{x1} = 0 & \quad E_{z1} = 0 \quad x \in (-1, +1) \\ H_{x1} = H_{x2} & \quad H_{z1} = H_{z2} \quad x \in (-\infty, -1) \cup (+1, +\infty). \end{aligned} \quad (4)$$

III. THE INTEGRAL EQUATIONS OF THE PROBLEM

By imposing the conditions (4), we get the following equations:

$$\int_0^\infty A(k) \cos(kx) dk = 0, \quad x \in (-1, +1) \quad (5)$$

$$\begin{aligned} \int_0^\infty \{a_1(k)A(k) + b_1(k)B(k)\} \sin(kx) dk = & \frac{\omega \mu}{2\beta k_0^2} I_z^0 \operatorname{sgn}(x), \\ x \in (-\infty, -1) \cup (+1, +\infty) \end{aligned} \quad (6)$$

$$\int_0^\infty B(k) \cos(kx) dk = V_0 / k_0^2, \quad x \in (-1, +1) \quad (7)$$

$$\begin{aligned} \int_0^\infty \{a_2(k)A(k) + b_2(k)B(k)\} \sin(kx) dk = 0, \\ x \in (-\infty, -1) \cup (+1, +\infty). \end{aligned} \quad (8)$$

In (6), $\operatorname{sgn}(x)$ is the sign function and we have also denoted

$$a_1(k) = \left[\left(\frac{1}{\alpha_1} - \frac{\alpha_1}{\beta^2} \right) \coth(\alpha_1 h_1) + \left(\frac{1}{\alpha_2} - \frac{\alpha_2}{\beta^2} \right) \coth(\alpha_2 h_2) \right] / k \quad (9)$$

$$a_2(k) = b_1(k) = \frac{k}{\alpha_1} \coth(\alpha_1 h_1) + \frac{k}{\alpha_2} \coth(\alpha_2 h_2) \quad (10)$$

$$\begin{aligned} b_2(k) = & (k_0^2 \epsilon_1 - \beta^2) \frac{k}{\alpha_1} \coth(\alpha_1 h_1) \\ & + (k_0^2 \epsilon_2 - \beta^2) \frac{k}{\alpha_2} \coth(\alpha_2 h_2). \end{aligned} \quad (11)$$

The equations given by x components in (4) were integrated with respect to the x variable, and I_z^0 and V_0 are two integration constants. In fact, I_z^0 is the current flowing in the strip in the direction of the z axis.

We perform now the following change of variables:

$$C(k) = a_1(k)A(k) + b_1(k)B(k) \quad (12)$$

$$D(k) = a_2(k)A(k) + b_2(k)B(k) \quad (13)$$

The resulting equation can be written in the form

$$\begin{aligned} \int_0^\infty C(k) \cos(kx) dk = & 2V_0 / k_0^2 + \int_0^\infty \{c_1(k)C(k) \\ & + d_1(k)D(k)\} \cos(kx) dk, \\ x \in (-1, +1) \end{aligned} \quad (14)$$

$$\begin{aligned} \int_0^\infty C(k) \sin(kx) dk = & \frac{\omega \mu}{\beta k_0^2} I_z^0 \operatorname{sgn}(x), \\ x \in (-\infty, -1) \cup (+1, +\infty) \end{aligned} \quad (15)$$

$$\begin{aligned} \int_0^\infty D(k) \cos(kx) dk = & (\epsilon_1 + \epsilon_2 - 2\bar{\beta}^2) V_0 + \int_0^\infty \{C_2(k)C(k) \\ & + d_2(k)D(k)\} \cos(kx) dk, \\ x \in (-1, +1) \end{aligned} \quad (16)$$

$$\begin{aligned} \int_0^\infty D(k) \sin(kx) dk = 0, \quad x \in (-\infty, -1) \cup (+1, +\infty). \\ (17) \end{aligned}$$

In the above relations, we have put

$$c_1(k) = 1 + \{2\beta^2 a_2(k) + 2b_2(k)\} / \Delta / \beta^2 \quad (18)$$

$$d_1(k) = -\{2\beta^2 a_1(k) + 2b_1(k)\} / \Delta / \beta^2 \quad (19)$$

$$c_2(k) = -\{[2\beta^2 - k_0^2(\epsilon_1 + \epsilon_2)]a_2(k) + 2b_2(k)\} / \Delta \quad (20)$$

$$d_2(k) = 1 - \{[k_0^2(\epsilon_1 + \epsilon_2) - 2\beta^2]a_1(k) - 2b_1(k)\} / \Delta \quad (21)$$

$$\Delta = a_1(k)b_2(k) - a_2(k)b_1(k). \quad (22)$$

It can be verified directly that we have:

$$c_j(k) = O(k^{-2}) \quad d_j(k) = O(k^{-2}) \quad \text{for } k \rightarrow \infty \ (j = 1, 2). \quad (23)$$

The relations (14)–(17) constitute the system of integral equations for solving the problem. Assuming, for the moment, that the right-hand-side terms are known, we solve the systems of coupled integral equations (14), (15) and (16), (17). A system of dual integral equations of the same type was considered in [18] in solving the microstrip problem in the static case. Taking account of those considered in the above-mentioned paper, we

can write directly the solutions

$$\tilde{C}(k) = \frac{1}{2\pi} \{ J_0(k)/k + 2c\delta(k) \} - 4 \sum_{n=1}^{\infty} n J_{2n}(k)/k \tilde{x}_n^{(1)} \quad (24)$$

$$\tilde{D}(k) = -4 \sum_{n=1}^{\infty} n J_{2n}(k)/k \tilde{x}_n^{(2)}. \quad (25)$$

Here we have put

$$C(k) = \frac{\omega\mu}{\beta k_0^2} I_z^0 \tilde{C}(k) \quad D(k) = \frac{\omega\mu}{\beta k_0^2} I_z^0 \tilde{D}(k). \quad (26)$$

The constants

$$\tilde{x}_n^{(j)} = \int_0^{\infty} \{ c_j(k) \tilde{C}(k) + d_j(k) \tilde{D}(k) \} J_{2n}(k) dk, \quad j = 1, 2; n = 0, 1, 2, \dots \quad (27)$$

are the solutions of the following double infinite systems of linear equations:

$$\begin{aligned} \tilde{x}_r^{(1)} &= \sum_{n=1}^{\infty} \{ \tilde{c}_{rn}^{(1)} \tilde{x}_n^{(1)} + \tilde{d}_{rn}^{(1)} \tilde{x}_n^{(2)} \} + \tilde{b}_r^{(1)} \\ \tilde{x}_r^{(2)} &= \sum_{n=1}^{\infty} \{ \tilde{c}_{rn}^{(2)} \tilde{x}_n^{(1)} + \tilde{d}_{rn}^{(2)} \tilde{x}_n^{(2)} \} + \tilde{b}_r^{(2)}. \end{aligned} \quad (28)$$

We have denoted

$$\tilde{c}_{rn}^{(j)} = -4n \int_0^{\infty} c_j(k) J_{2r}(k) J_{2n}(k) / k dk \quad (29)$$

$$\tilde{d}_{rn}^{(j)} = -4n \int_0^{\infty} d_j(k) J_{2r}(k) J_{2n}(k) / k dk \quad (j = 1, 2) \quad (30)$$

$$\begin{aligned} \tilde{b}_0^{(1)} &= \frac{1}{\pi} \left\{ \int_1^1 [J_0^2(k) \cdot c_1(k) - 1] / k dk \right. \\ &\quad \left. + \int_1^{\infty} J_0^2(k) c_1(k) / k dk + c \right\} \end{aligned} \quad (31)$$

$$\tilde{b}_0^{(2)} = \frac{1}{\pi} \int_0^{\infty} c_2(k) J_0^2(k) / dk \quad (32)$$

$$\tilde{b}_r^{(j)} = \frac{1}{\pi} \int_0^{\infty} c_j(k) J_0(k) J_{2r}(k) / k dk \quad (j = 1, 2; r = 1, 2, \dots) \quad (33)$$

$c = 0.577216 \dots$ being Euler's constant.

Beside these, in order to obtain the above solutions the following compatibility relations must be fulfilled:

$$\left\{ \frac{\omega\mu}{\beta} \left[\frac{1}{\pi} \ln 2 - \tilde{x}_0^{(1)} \right] I_z^0 - 2V_0 = 0 \right. \quad (34)$$

$$\left. \frac{\omega\mu}{\beta k_0^2} \tilde{x}_0^{(2)} I_z^0 + (\epsilon_1 + \epsilon_2 - 2\beta^2) V_0 = 0. \right. \quad (35)$$

In fact we have two relations relating the constants I_z^0 and V_0 . The compatibility condition is

$$\left(k_0^2 \frac{\epsilon_1 + \epsilon_2}{2} - \beta^2 \right) \left(\frac{1}{\pi} \ln 2 - \tilde{x}_0^{(1)} \right) + \tilde{x}_0^{(2)} = 0 \quad (36)$$

and it is exactly the desired dispersion equation of the problem.

The double infinite set of equations (28) and (29) for index values $r = 1, 2, \dots$ determines the constants $\tilde{x}_n^{(j)}, \tilde{x}_2^{(j)}, \dots$ and the first relation in each set gives the values of the constants $\tilde{x}_0^{(j)}$ ($j = 1, 2$). Moreover, the dispersion relation (36) gives the values of the propagation constant.

TABLE I

Frequency (GHz)	2×2 Matrix [10]	4×4 Matrix [10]	100×100 Matrix [6]	Eq. (36)
10	0.530	0.531	0.55	0.566
20	1.10	1.115	1.17	1.1699
30	1.71	1.74	1.77	1.787

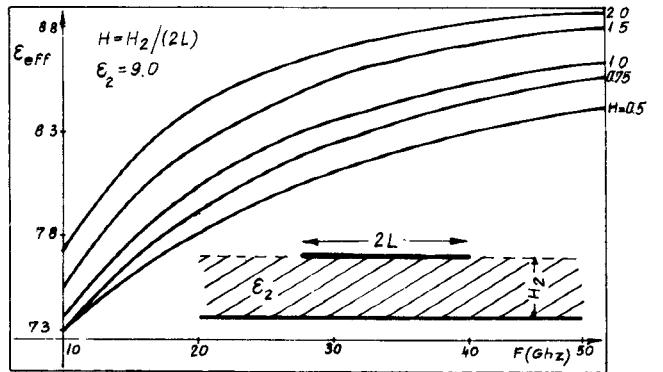


Fig. 2. Effective dielectric constant versus frequency.

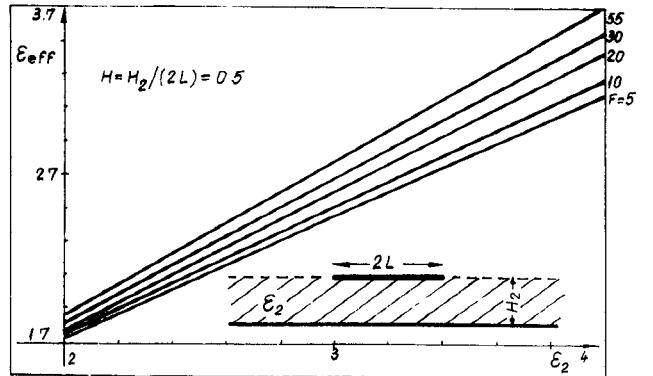


Fig. 3. Effective dielectric constant versus substrate dielectric constant.

IV. APPLICATIONS

We applied the formulas given above to determine the dispersive characteristic of certain microstrip structures.

The first example is the structure having the following parameters: $\epsilon_1 = 1, \epsilon_2 = 9.0, h_1 = 3, h_2 = 1$. By the above normalization these are just the parameters considered in [6] and [10]. (In fact the completely shielded structure used in the above mentioned papers also has two lateral sides for $x = \pm 3.5$.)

In system (28) we considered only the equations corresponding to $r = 1$ and have put $\tilde{x}_n^{(j)} = 0$ for $n \geq 2$. The results obtained are given in Table I. They agree very well with the parameters obtained in the studies mentioned above, although our structure is slightly different from the completely shielded structure they used.

We have also studied, for the open microstrip line, the dependence of the effective dielectric constants upon the dielectric thickness and also on the relative dielectric constant for various values of frequency. The results are plotted in Fig. 2 and Fig. 3. Note the linear variance of the effective dielectric constant with respect to dielectric permittivity for every frequency.

We emphasize the good convergence properties of the infinite set of linear equations. As in the static analysis [18], it is sufficient to consider in the infinite system only a single equa-

tion in order to obtain accurate numerical results. We can also obtain, by means of (36), the higher order modes. In this case the integrals which express the equation coefficients must be regarded as principal values since function $\Delta(k)$ vanishes inside the integration domain.

V. CONCLUSIONS

A new method is given for calculating the dispersion characteristics of microstrip lines. The analysis is rigorous and it expresses the solution of the dispersion equation in terms of the solution of a double infinite system of linear equations. The system coefficients are given by certain quadratures. The numerical examples reveal the high convergence order of the method.

REFERENCES

- [1] H. A. Wheeler, "Transmission-line properties of parallel strips separated by dielectric sheet," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-13, pp. 172-185, Mar 1965.
- [2] E. Yamashita and R. Mittra, "Variational method for the analysis of microstrip lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-16, pp. 251-256, Apr. 1968.
- [3] P. Silvester, "TEM wave properties of microstrip transmission lines," *Proc. Inst. Elec. Eng.*, vol. 115, pp. 43-48, Jan. 1968.
- [4] D. Homencovschi, A. Manolescu, A. M. Manolescu, and L. Kreindler, "An analytical solution for the coupled stripline-like microstrip line problem," *IEEE Trans. Microwave Theory Tech.*, vol. 36, pp. 1002-1007, June 1988.
- [5] E. J. Denlinger, "A frequency dependent solution for microstrip transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-19, pp. 30-39, Jan. 1971.
- [6] J. S. Hornsby and A. Gopinath, "Numerical analysis of a dielectric-loaded waveguide with a microstrip line—Finite difference method," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-17, pp. 684-690, Sept. 1969.
- [7] P. Daly, "Hybrid-mode analysis of microstrip by finite element methods," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-19, pp. 19-25, Jan. 1971.
- [8] G. I. Zysman and D. Varon, "Wave propagation in microstrip transmission lines," in *1969 IEEE MTT-S Int. Microwave Symp. Dig.* (Dallas, TX), pp. 3-9.
- [9] E. Yamashita and K. Atuski, "Analysis of microstrip-like transmission lines by nonuniform discretization of integral equations," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-24, pp. 195-200, Apr. 1976.
- [10] R. Mittra and T. Itoh, "A new technique for the analysis of the dispersion characteristics of microstrip lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-19, pp. 47-56, Jan. 1971.
- [11] T. Itoh and R. Mittra, "Spectral domain approach for calculating the dispersion characteristics of microstrip-lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-21, pp. 496-499, July 1973.
- [12] M. Kobayashi and F. Ando, "Dispersion characteristics of open microstrip lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-35, p. 105, Feb. 1987.
- [13] N. Fache and D. De Zutter, "Rigorous full-wave space domain solution for dispersive microstrip lines," *IEEE Trans. Microwave Theory Tech.*, vol. 36, pp. 731-737, Apr. 1988.
- [14] K. Uchida, T. Noda, and T. Tatsunga, "New type of spectral-domain analysis of a microstrip line," *IEEE Trans. Microwave Theory Tech.*, vol. 37, pp. 947-952, June 1989.
- [15] C. Shin, R. B. Wu, S. K. Jeng, and C. H. Chen, "A full wave analysis of microstrip lines by variational conformal mapping technique," *IEEE Trans. Microwave Theory Tech.*, vol. 36, pp. 576-581, Mar. 1988.
- [16] E. F. Kuester and D. C. Chang, "An appraisal of methods for computation of the dispersion characteristics of open microstrip," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-27, pp. 691-694, 1979.
- [17] R. F. Harrington, *Time-Harmonic Electromagnetic Fields*. New York: McGraw-Hill, 1961.
- [18] D. Homencovschi, "An analytical solution for the microstrip line problem," *IEEE Trans. Microwave Theory Tech.*, vol. 38, pp. 766-769, June 1990.

Admittance Calculation of a Slot in the Shield of a Multiconductor Transmission Line

Richard G. Plumb

Abstract—The admittance calculation for a narrow slot in the conducting shield of a multiconductor transmission line is presented. The admittance represents a generalized admittance resulting from an asymptotic, one-term moment method solution and is approximated using transmission line theory. The calculated admittance is useful in modeling connectors for multiconductor transmission lines. Some useful impedance calculations for multiconductor transmission lines are developed.

I. INTRODUCTION

In a recent paper [1], we presented a method of modeling connectors for multiconductor transmission lines (MTL's). In that investigation the connector was modeled as a narrow circumferential slot, of width d , in the shield of an MTL. The MTL, uniform in the axial direction and having an arbitrary cross section, contained N lines and a conducting shield of finite width t . The interior medium of the MTL was assumed lossless and homogeneous. The problem was solved by treating the slot as a thick aperture in the shield. The equivalence principle was invoked to obtain two coupled integral equations in the equivalent surface magnetic currents. A one-term moment method solution was then obtained for an electrically narrow slot and a small shield radius. The moment method solution led to an equivalent circuit representation. Power calculations were derived from the equivalent circuit for the power radiated through the slot and the power transmitted down the line. When the slot admittance is replaced by the transfer admittance of a connector, the power radiated through the slot becomes the power radiated through the connector.

The original MTL network and an equivalent circuit are shown in Fig. 1. The equivalent circuit consists of the admittances Y^a , Y^b , and Y^c , corresponding to the generalized admittance of the internal region of the MTL, the slot region, and the region external to the MTL respectively. For a one-term moment method solution, Y^c is the radiation admittance or the external input admittance of the antenna formed by the outer shield surface, having a finite feed width of length d , when a uniform electric field excites the antenna. The admittance Y^b corresponds to the transfer admittance of the slot, which can be interpreted as the transfer admittance of a connector. The current source I^i is a generalized current source and is obtained by calculating the net short circuit shield current on the inner shield surface. The current I' is the current on the inner shield surface when the slot is covered with a perfect conductor.

In this paper we detail the calculations for the admittance Y^a . An approximate expression for the admittance is obtained using transmission line theory. In doing so, some useful impedance calculations for multiconductor transmission lines are presented.

II. STATEMENT OF THE PROBLEM

The admittance Y^a is the admittance at the inner slot surface looking into the MTL when a uniform magnetic current is placed over the shorted surface [2]. The magnitude of the

Manuscript received May 17, 1990; revised November 19, 1990.

The author is with the Department of Electrical and Computer Engineering, University of Kansas, Lawrence, KS 66045-2969.
IEEE Log Number 9042497.